Transmission Systems

New joint frame synchronisation and carrier frequency offset estimation method for OFDM systems†

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SUMMARY

We propose a new joint frame synchronisation and carrier frequency offset estimation scheme for burst transmission mode OFDM systems. This scheme uses a central-symmetric and comb-like (CSCL) training sequence, which eases the power detection at the receiver without increasing the total training sequence power. Fine frame synchronisation as well as carrier frequency offset acquisition with a maximum acquisition range of \( \pm \frac{N}{4} \times SF \) times the sub-carrier spacing can also be performed based on the proposed CSCL training sequence, where \( N \) is the discrete Fourier transform (DFT) length and \( SF \) is an integer-valued spreading factor used to generate CSCL. The post-acquisition residual carrier frequency offset can be further estimated and corrected via a fine adjustment algorithm. In order to reduce performance loss due to the high peak-to-average power ratio (PAPR) of the CSCL training sequence, a time-domain constant-envelope (CE) training sequence is also proposed. The superior estimation accuracy of the proposed algorithm over that of the Moose algorithm and the SS (Shi and Serpedin) algorithm is proved by computer simulation.

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1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) effectively combats multipath fading [1, 2]. Moreover, the cyclic prefix (CP) in OFDM mitigates inter-symbol interference (ISI). OFDM has been adopted by DAB (digital audio broadcasting) [3], ADSL (asymmetry digital subscriber loop) [4] and the IEEE802.11a standard at the 5-GHz band with a data rate up to 54 Mb/s [5]. OFDM can also be combined with CDMA (code-division multiple-access), such as multicarrier-CDMA (MC-CDMA) and multicarrier DS-CDMA [6]. In a broadband channel with an approximate 50–100 MHz bandwidth, variable spreading factor-orthogonal frequency and code division multiplexing (VSF-OFCDM) has been proposed by NTT DoCoMo. This technology is based on MC-CDMA for the forward link [7].

Many frame synchronisation algorithms for OFDM systems are available [8–19]. The algorithm [11] performs synchronisation based on the CP, resulting only in symbol level synchronisation. Null sub-carriers can also be utilised to find the beginning of a frame [20], but this approach is unsuitable for burst transmission modes. Some estimators, either training-symbol-aided [14, 15] or not [11–13], correlate the two parts of a received training sequence to perform synchronisation. Using a training sequence consisting of two identical blocks, Schmidl and Cox [15] proposed a timing synchronisation algorithm, which searches for the maximum correlation between these two...
blocks. A drawback of this algorithm is that a plateau in the peak of the timing metric will cause a large variance error. Shi and Serpedin (SS) [21] improved the synchronisation performance by adopting a training structure [B B -B B]. With a sharper timing metric, this improved algorithm is robust to ISI, and, as a result, SS guarantees a smaller synchronisation error than that of the Schmidl algorithm. However, the timing metric also appears as a large value at a position neighbouring the location of the start of the training sequence (the correct synchronisation position). This value degrades its synchronisation performance at a low signal-to-noise ratio (SNR). Since all these existing algorithms use training sequences with the same average power as that of data symbols, high symbol miss detection probability may occur when they are used in a burst transmission system.

OFDM systems are also sensitive to the carrier frequency offset. Some conventional carrier frequency offset estimation algorithms transmit periodic identical patterns. By measuring phase differences between these patterns at the receiver, the carrier frequency offset can be estimated [14, 15, 18, 22–26]. The estimator proposed in Reference [11] does not need a special training sequence. This estimator uses only the CP to estimate the carrier frequency offset, but the estimation range is limited within ±0.5 sub-carrier spacing. By utilizing virtual sub-carriers, high performance carrier frequency offset estimators can also be designed [20, 27]. In References [15, 22], some algorithms proposed for increasing the acquisition range, but in Reference [15], two training symbols are needed. In Reference [22], shortened training symbols are used for acquisition; however, the estimation accuracy is degraded.

In this paper, a new joint frame synchronisation and carrier frequency offset estimation scheme for OFDM is proposed. The essence of the proposed scheme is as follows: A central-symmetric and comb-like (CSCL) training sequence is first generated at the transmitter by using an adjustable spreading factor—SF. At the receiver, for a pre-defined false alarm probability (the receiver falsely sees noise as an incoming signal because of the high noise power), a well-defined power-detection algorithm is provided to detect a new incoming training sequence correctly with a low miss detection probability (the receiver does not detect a new incoming frame because of the low SNR). After power detection, a joint fine frame synchronisation and carrier frequency offset acquisition algorithm is performed. Based on this algorithm, the boundary of the training sequence/frame can be found and the carrier frequency offset can be coarsely estimated simultaneously. The maximum carrier frequency offset acquisition range in the proposed scheme is up to ±N/(4×SF) times the sub-carrier spacing. Finally, a new maximum-likelihood (ML) carrier frequency offset fine adjustment algorithm is also proposed to estimate and correct the remaining carrier frequency offset after acquisition. In order to resolve the high peak-to-average power ratio (PAPR) problem in the CSCL training sequence with large SF, we also propose a time-domain Constant-Envelope (CE) training sequence in this paper, and considerable performance improvement over the non-CE CSCL based algorithm is achieved in the CE-based algorithm in a PAPR limited system.

The remainder of this paper is organised as follows. In Section 2, the system’s details are introduced. In Section 3, a new power detection and coarse frame synchronisation algorithm based on the proposed Comb-like training symbol is proposed and analysed. A new CSCL-based joint fine frame synchronisation and carrier frequency offset acquisition algorithm is given in Section 4, and an ML carrier frequency offset fine adjustment algorithm based on the same training sequence is proposed in Section 5. A time-domain CE training sequence is proposed in Section 6. Section 7 provides the simulation results. Finally, Section 8 concludes the paper.

Notation: (·)T and (·)∗ denote transpose and complex conjugate, respectively. The imaginary unit is \( j = \sqrt{-1} \). \( \mathbb{E}[x] \) and \( \text{Var}[x] \) denote the mean and the variance of \( x \), respectively. A circularly symmetric complex Gaussian variable with mean \( m \) and variance \( \sigma^2 \) is denoted by \( z \sim \mathcal{CN}(m, \sigma^2) \).

## 2. SYSTEM FUNDAMENTALS

In conventional OFDM systems, input symbols are chosen from a complex signal constellation, e.g. phase-shift keying (PSK) or quadrature amplitude modulation (QAM). Let \( X_k \) \( (k = 0, 1, \ldots, N - 1) \) be the complex input symbol modulating the \( k \)th sub-carrier. The time-domain received signal samples, \( y(n) \), are related to the input symbols via an inverse discrete Fourier transform (IDFT), given by

\[
y(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{j \frac{2\pi nk}{N}} + w(n)
\]

where \( v(n) \) is the IDFT output (signal part), and \( w(n) \) is an additive white Gaussian noise (AWGN) term, of zero mean and variance \( \sigma^2_w = \mathbb{E}[(w(n))^2] \). It is known that \( y(n) \) is a stationary Gaussian process. The average SNR at the

\[413–430\]

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The proposed synchronisation scheme is performed based on a CSCL training sequence. This sequence is composed of two time-domain concatenated training symbols, where the second training symbol is the reverse replica of the first. The first training symbol is generated by constructing a length-$N_{\text{SF}}$ frequency-domain sub-block with structure $\{X_0, X_1, \ldots, X_{SF-1}\}$, where SF is a predetermined spreading factor. After frequency-domain spreading, each sub-block is replicated SF times to form the length-$N$ frequency-domain sequence:

$$x = \{X_0, X_1, \ldots, X_{N/SF-1}\} \underbrace{X_0, X_1, \ldots, X_{N/SF-1}}_{\text{sub-block}_1} \underbrace{X_0, X_1, \ldots, X_{N/SF-1}}_{\text{sub-block}_2} \ldots \underbrace{X_0, X_1, \ldots, X_{N/SF-1}}_{\text{sub-block}_{SF}}$$  \hfill (3)

as shown in Figure 1. The IDFT output of $x$ is a time-domain comb-like training symbol:

$$v(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{\frac{j2\pi nk}{N}} = \frac{\sin(n\pi)}{N \cdot \sin\left(\frac{n\pi}{SF}\right)} e^{\frac{j\pi SF(n-1)}{N}} \sum_{k=0}^{N/SF-1} X_k \cdot e^{\frac{j2\pi nk}{N}},$$  \hfill (4)

where $\sigma^2 = \mathbb{E}\{ | \sum_{k=0}^{N-1} X_k \cdot e^{\frac{j2\pi nk}{N}} |^2 \}$.

Figure 1. Joint frame synchronisation and carrier frequency offset estimation in burst transmission mode multicarrier systems.

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When $N/SF$ is large enough, the central limit theorem (CLT) [28, pp. 59–61] suggests that $v(n)$ can be approximated as a zero-mean complex Gaussian process of variance:

$$
\sigma_v^2 = \mathbb{E}\left\{ \left( \frac{\sin(n\pi)}{SF \cdot \sin\left(\frac{n\pi}{SF}\right)} \right)^2 \cdot \frac{SF}{N} \cdot \sum_{k=0}^{N/SF-1} X_k \cdot e^{j2\pi k} \right\}^2
$$

$$
= \left| \frac{\sin(n\pi)}{SF \cdot \sin\left(\frac{n\pi}{SF}\right)} \right|^2 \cdot SF \cdot \sigma_s^2
$$

(5)

Since $\lim_{n=m \times SF} \mathbb{E}[|v(n)|^2] = SF \cdot \sigma_s^2$, and $\lim_{n \neq m \times SF} \mathbb{E}[|v(n)|^2] = 0$ where $m = 0, 1, \ldots, N/SF - 1$, we conclude that by transmitting $v(n)$, the received SNR of $v(n)$'s non-zero samples can be increased SF times without changing the symbol's total power.

### 3. POWER DETECTION AND COARSE FRAME SYNCHRONISATION

Power-detection performance at the receiver is characterised by the false alarm probability ($-P_{FA}$), the probability that noise is falsely seen as a signal. By comparing the total power of $L(1 \leq L \leq N/SF)$ continuously received samples, i.e. $\Omega(n, L) = \sum_{m=0}^{L-1} |y(n + m \times SF)|^2$, with a decision threshold $L \times \Gamma$ ($\Gamma$ is a pre-specified threshold), we will know that a new symbol has been detected once the tested power is higher than threshold. It is reasonable to formulate $\Omega(n, L)$ as a central $\chi^2$ random variable with $2L$ degrees of freedom [28]. The probability that $\Omega(n, L)$ is smaller than $L \times \Gamma$ (miss detection probability) can be derived as

$$
P_{MD}^L = P(\Omega(n, L) < L \cdot \Gamma) = 1 - e^{-\frac{L}{\sigma_w^2 + \sigma_w^2}} \cdot \sum_{i=0}^{L-1} \frac{L \cdot \Gamma}{i!} \left( \frac{L \cdot \Gamma}{\sigma_w^2 + \sigma_w^2} \right)^i
$$

(6)

Since $w(n) \sim \mathcal{CN}(0, \sigma_w^2)$, $W = |w(n)|^2$ is a central $\chi^2$ random variable with 2 degrees of freedom, and its probability density function (pdf) is given by

$$
f_W(w) = \frac{1}{\sigma_w^2} \cdot \exp\left\{-\frac{w}{\sigma_w^2}\right\}, \quad w \geq 0
$$

(7)

The false alarm probability ($-P_{FA}$) is given by $P_{FA} = \int_0^{\infty} f_W(w) \, dw = \exp\left\{-\frac{1}{\sigma_w^2}\right\}$. For a given pre-defined $P_{FA}$, $\Gamma$ can be determined by $\Gamma = \sigma_w^2 \cdot \ln\left(\frac{1}{P_{FA}}\right)$, so that Equation (6) can be rewritten as

$$
P_{MD}^L = 1 - e^{-\frac{L}{\sigma_w^2 + \sigma_w^2}} \cdot \sum_{i=0}^{L-1} \frac{L \cdot \Gamma}{i!} \left( \frac{L \cdot \Gamma}{\sigma_w^2 + \sigma_w^2} \right)^i
$$

$$
= 1 - (P_{FA})^{\frac{L}{\sigma_w^2 + \sigma_w^2}} \cdot \sum_{i=0}^{L-1} \frac{L \cdot \Gamma}{i!} \left( \frac{L \cdot \Gamma}{\sigma_w^2 + \sigma_w^2} \right)^i
$$

(8)

Compared to Equation (8), the miss detection probability in conventional OFDM systems (equivalent to the proposed scheme with SF = 1) is found to be

$$
P_{Con}^L = 1 - (P_{FA})^{\frac{L}{\sigma_w^2 + \sigma_w^2}} \cdot \sum_{i=0}^{L-1} \frac{L \cdot \Gamma}{i!} \left( \frac{L \cdot \Gamma}{\sigma_w^2 + \sigma_w^2} \right)^i
$$

(9)

where $1 \leq L \leq N$.

The proposed comb-like training symbol improves the power detection compared to the conventional symbols (SF = 1), as illustrated in Figure 2. In the proposed scheme, a larger SF implies higher reliability in power detection, and its advantage over conventional schemes becomes more pronounced as the SNR decreases. Note that Equation (8) holds for an AWGN channel, and this detection performance cannot be achieved if the multipath channels are considered, because in the proposed training symbols, non-zero samples are sparsely distributed. When

![Figure 2](image-url)
they are transmitted through a multipath channel, if SF is large enough, non-zero samples from different taps usually do not overlap. Thus, each tap can be logically seen as an AWGN channel, and the power detection of the proposed training symbol is tap by tap. Since multipath fading reduces the SNR of each tap, the power-detection performance will degrade compared to that in the AWGN channel. The power of different taps will add up at the receiver if conventional symbols are transmitted through a multipath channel, and this superposition prevents a heavy degradation in power-detection performance. However, the proposed training symbol is expected to have advantages over conventional ones in terms of power detection, although these advantages may not be as remarkable as that achieved in the AWGN channel. For example, for \( N = 64 \) and \( P_{FA} = 10^{-3} \), consider a multipath channel given in Table 1. For an averaged SNR of 10 dB, if SF = 8, a miss detection probability of \( 2 \times 10^{-3} \) can be achieved with the proposed training symbols by detecting only four consecutive non-zero samples in Tap 1. This probability is lower than the miss detection probability of \( 8 \times 10^{-3} \) achieved with conventional symbols by detecting 32 continuous samples, as illustrated in Figure 2.

Although the proposed training sequence outperforms conventional symbols, its synchronisation may fail if one or more non-zero samples are mis-sampled, especially for large SF. To avoid this risk, over-sampling should be performed at the receiver. If the over-sampling frequency is high enough (e.g. double or four times the signal rate), each non-zero sample can be correctly sampled without missing any samples.

The proposed training sequence is especially suitable for a burst transmission mode system. Some conventional algorithms, e.g. Reference [15], perform frame detection by exploiting the correlativity in the training sequence. Frame detection as well as fine frame synchronisation will be performed once a local peak of the timing metric is found [15]. The performance of this kind of frame detection algorithm is robust in either burst or continuous transmission mode systems. Figure 3 compares the frame miss detection probability of the proposed method and the Schmidl algorithm [15]. The frame miss detection probability in the proposed algorithm can be explained as follows: a new coming frame is assumed to be detected if one or more samples in the proposed training symbol are detected, and miss-detection of a frame appears only when no training sample is detected. Thus, the miss detection probability of a total frame should be much lower than that representing a part of its samples. From Figure 3, we know that at low SNR, the Schmidl algorithm outperforms the proposed scheme with respect to a lower frame miss detection probability, and the proposed algorithm will achieve its advantages with an increase of SNR. This result seems to contradict Figure 2 which indicates that the proposed scheme outperforms the conventional ones at a low SNR. This contradiction occurs because Figure 2 compares the power-detection performance between the proposed training symbol and the conventional symbols based on Equation (8), where power detection is sample by sample, and the detected samples are assumed to be independent and identically distributed (i.i.d.) Gaussian random variables. However, Figure 3 considers the detection of a total frame. In the Schmidl algorithm, correlativity in the training sequence is exploited when performing frame detection, and the conjugate correlation operation in the timing metric randomises the noise or interference greatly. This noise/interference reduction capability makes the correlativity-exploiting frame detection method outperform the proposed power-concentration method at low SNR.

**Table 1. Multipath wireless channel scenarios.**

<table>
<thead>
<tr>
<th>Carrier</th>
<th>5-GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Taps delay (samples)</td>
<td>0 2 3 6</td>
</tr>
<tr>
<td>Channel attenuation (dB)</td>
<td>0 -3 -6 -9</td>
</tr>
<tr>
<td>Initial phase distribution</td>
<td>([0,2\pi]) ([0,2\pi]) ([0,2\pi]) ([0,2\pi])</td>
</tr>
</tbody>
</table>

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In real systems with continuous transmission modes, NULL symbols are usually used for power detection [16]. By periodically inserting the NULL symbol into each frame, a new incoming frame can be identified at the receiver by detecting the NULL symbol (at continuously low power). Figure 4 compares the power-detection performance of the proposed method and the NULL-symbol-aided method [16]. A multipath fading channel presented in Table 1 is considered. In each algorithm, the total symbol length for power detection is 64. In real systems, adaptively adjusting the power-detection threshold according to the current channel condition to guarantee a fixed false alarm probability is complicated or even impossible. A feasible method is to carefully select a threshold and keep it unchanged during the course of power detection, no matter what the SNR is. The threshold is usually set to some percentage of the averaged signal power, e.g. $\Gamma = 0.1 \times \sigma_s^2$. Once a threshold is selected this way, the miss detection probability in the proposed scheme will not change too much as the SNR increases, as shown in Figure 4. For a given signal power, a higher SNR implies a comparably lower noise power, which reduces the miss detection probability in the Santella algorithm. Figure 4 shows that the power-concentration property makes the proposed algorithm outperform the Santella algorithm with respect to a lower frame miss detection probability. As $\Gamma$ increases, this performance advantage will be reduced accordingly. For example, when we set $\Gamma = 0.2 \times \sigma_s^2$, the proposed algorithm with SF = 8 has a performance improvement of 7.5 dB over the Santella algorithm, and this advantage will be reduced to 3.2 dB if we raise $\Gamma$ to $0.5 \times \sigma_s^2$.

4. JOINT FINE FRAME SYNCHRONISATION AND CARRIER FREQUENCY OFFSET ACQUISITION

As discussed in Section 3, the power-detection performance is considerably improved if the proposed comb-like training symbol is used. After power detection, a coarse timing synchronisation can also be achieved simultaneously. However, power detection indicates only the arrival of a new symbol, without pointing out its exact starting point. In order to demodulate the transmit signal without introducing ISI, an accurate fine frame synchronisation algorithm is needed.

In this section, a joint algorithm is proposed, which performs fine frame synchronisation and carrier frequency offset acquisition simultaneously, as illustrated in Figure 5. At the start, the receiver should be switched to ‘1’ to perform power detection. Coarse frame synchronisation is achieved as soon as a new training sequence is detected. All the detected samples should be buffered to perform joint fine frame synchronisation and carrier frequency offset acquisition. Next, the switch should be changed to ‘2’, and then the process of fine synchronisation starts.

Given a normalised carrier frequency offset $\varepsilon$, the relationship between a transmitted sample and its received counterpart is

$$y(n) = v(n - \theta) \cdot e^{j\frac{2\pi n}{N}} + w(n), \quad n \geq 0$$  \hspace{1cm} (10)
then be performed as synchronisation and carrier frequency offset acquisition can be illustrated in Figure 6.) We name this new training sequence the second one as \( v(0) \) is the reverse repeat of the first one. (After generating many conventional fine frame synchronisation algorithms discussed in Section 3), and the second training symbol is the reverse repeat of the first one. (After generating the first training symbol as \( \{v(0), v(1), \ldots, v(N - 1)\} \), the second one is \( \{v(N - 1), v(N - 2), \ldots, v(0)\} \), as illustrated in Figure 6.) We name this new training sequence the CSCL training sequence. The proposed joint fine frame synchronisation and carrier frequency offset acquisition can then be performed as

\[
\{\hat{\theta}; \hat{\varepsilon}\} = \arg \max_{\{\theta; \varepsilon\}} \Phi_{\theta}(\varepsilon)
\]  

(11)

where \( \Phi_{\theta}(\varepsilon) = \left| \sum_{n=0}^{N/\text{SF} - 1} \Psi_{\theta}(n) \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} \right|^2 \) with \( \Psi_{\theta}(n) = y(i + 2N - 1 - n \cdot \text{SF}) \cdot y^*(i + n \cdot \text{SF}) \) and \( n \in [0, N/\text{SF} - 1] \).

We briefly analyse the statistical properties of \( \Psi_{\theta}(n) \). When \( i = \theta \) (correct timing synchronisation is assumed), \( \Psi_{\theta}(n) \) can be represented as

\[
\Psi_{\theta}(n) = y(\theta + 2N - 1 - n \cdot \text{SF}) \cdot y^*(\theta + n \cdot \text{SF})
\]

\[= v(2N - 1 - n \cdot \text{SF}) \cdot y^*(n \cdot \text{SF}) \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} + v(2N - 1 - n \cdot \text{SF}) \cdot w^*(\theta + n \cdot \text{SF}) + w(\theta + 2N - 1 - n \cdot \text{SF}) \cdot y^*(n \cdot \text{SF}) \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} + w(\theta + 2N - 1 - n \cdot \text{SF}) \cdot w^*(\theta + n \cdot \text{SF})
\]

\[= |v(n \cdot \text{SF})|^2 \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} + \eta_n^\theta
\]  

(12)

where \( \eta_n^\theta \) denotes the noise-related items of \( \Psi_{\theta}(n) \). Based on it, \( \Phi_{\theta}(\varepsilon) \) can be decomposed into

\[
\Phi_{\theta}(\varepsilon) = \left| \sum_{n=0}^{N/\text{SF} - 1} \Psi_{\theta}(n) \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} \right|^2
\]

\[= \sum_{n=0}^{N/\text{SF} - 1} |v(n \cdot \text{SF})|^2 \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} + \eta_n^\theta \cdot e^{j \frac{2\pi n \varepsilon}{\text{SF}}} \]  

(13)

When \( N/\text{SF} \) is large enough, \( \Phi_{\theta}(\varepsilon) \) can be approximated as a non-central \( \chi^2 \) random variable with 2 degrees of freedom, and its pdf is given by [28]:

\[
f_\Phi(\phi|\theta; \varepsilon) = \frac{1}{\sigma^2} \cdot e^{-\frac{\phi^2}{\sigma^2}} \cdot I_0 \left( \sqrt{\frac{2K}{\sigma^2}} \right)
\]

(14)

where \( \sigma^2 = \mathbb{E}[|\eta_n^\theta|^2] = 2\sigma_v^2 \cdot \sigma_w^2 + \sigma^2 \), \( K = \sigma_v^2 \cdot \frac{\sin^2 2\pi(\theta - \varepsilon)}{\sin^2 2\pi(\theta + \varepsilon)} \), and \( I_0(\bullet) \) is the zeroth-order modified Bessel function of the first kind. The mean of \( \Phi_{\theta}(\varepsilon) \) is

\[
\mathbb{E}[\Phi_{\theta}(\varepsilon)] = \sigma_v^4 \cdot \frac{\sin^2 2\pi(\theta - \varepsilon)}{\sin^2 2\pi(\theta - \varepsilon)/\text{SF}} + \sigma_n^2
\]

(15)

and its variance is derived as

\[
\text{Var}[\Phi_{\theta}(\varepsilon)] = 2\sigma_n^2 \cdot \sigma_v^4 \cdot \frac{\sin^2 2\pi(\theta - \varepsilon)}{\sin^2 2\pi(\theta + \varepsilon)/\text{SF}} + \sigma_n^4
\]

(16)

Equation (15) shows that \( \mathbb{E}[\Phi_{\theta}(\varepsilon)] \) is a periodic function of \( \varepsilon = \hat{\varepsilon} - \varepsilon \) (the carrier frequency offset estimation error) with period \( \frac{N}{\text{SF}} \). Within each period, we can uniquely determine \( \varepsilon \) from \( \hat{\varepsilon} \). Consequently, the maximum carrier frequency offset acquisition range of the proposed CSCL algorithm is as large as \( \pm \frac{N}{\text{SF}} \), times the sub-carrier spacings. When \( \hat{\varepsilon} = \varepsilon \), \( \mathbb{E}[\Phi_{\theta}(\varepsilon)] \) achieves its maximum value.

When \( i = \theta - k \times \text{SF} \), if \( N/\text{SF} \) is large enough, \( \Phi_{\theta - k \times \text{SF}}(\varepsilon) \) can be approximated as a central \( \chi^2 \) distributed random variable with 2 degrees of freedom, and its pdf is

\[
f_\Phi(\phi|\theta - k \times \text{SF}; \varepsilon) = \frac{1}{\sigma^2} \cdot e^{-\frac{\phi^2}{\sigma^2}}
\]

(17)

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where $\sigma^2 = \left(\frac{N}{SF} - 1 - k\right)\sigma_w^2 + 2\sigma_w^2 \sigma_w^4 + \sigma_w^4 + (k + 1)\left(\sigma_w^2 \sigma_w^2 + \sigma_w^4\right)$. We can easily derive the mean of $\Phi_{\theta-k\times SF}(\hat{\varepsilon})$ as $\mathbb{E}[\Phi_{\theta-k\times SF}(\hat{\varepsilon})] = \sigma^2$, and its variance as $\text{Var}[\Phi_{\theta-k\times SF}(\hat{\varepsilon})] = \sigma^4$.

In the proposed CSCL, any two adjacent non-zero samples are separated by $(SF - 1)$ contiguous zero samples. This separation makes the fine frame synchronisation error most likely to appear at $(\theta - k \times SF)$, with a negligible probability to appear at other positions. Figure 7 illustrates $\mathbb{E}[\Phi_{\hat{\theta}}(\hat{\varepsilon})]$ and $\mathbb{E}[\Phi_{\theta-k\times SF}(\hat{\varepsilon})]$ as functions of $e = \hat{\varepsilon} - \varepsilon$ (or $\hat{\varepsilon}$ for a given $\varepsilon$), where SNR = 10 dB and $N = 32$. Figure 7 shows that $\mathbb{E}[\Phi_{\hat{\theta}}(\hat{\varepsilon})]$ is a periodic function of $e = \hat{\varepsilon} - \varepsilon$ with period $\frac{N}{2 \times SF}$, and that it has a main lobe in each period. In contrast, $\mathbb{E}[\Phi_{\theta-k\times SF}(\hat{\varepsilon})]$ is a constant function independent of $e$. Within each period, a small area exists in the centre of the main lobe to satisfy $\Phi_{\hat{\theta}}(\hat{\varepsilon}) > \Phi_{\theta-k\times SF}(\hat{\varepsilon})$. In each main lobe, $\Phi_{\hat{\theta}}(\hat{\varepsilon})$ increases as $\varepsilon$ approaches to the main lobe’s centre. By exploiting this property, a fast joint fine frame synchronisation and carrier frequency offset acquisition can be performed.

Correct synchronisation with the proposed CSCL algorithm requires that timing synchronisation and carrier frequency offset acquisition be performed simultaneously. The oscillator of a new accessing terminal may not be perfectly calibrated with the base station; i.e. it has a large carrier frequency offset to be corrected (usually larger than 0.5). In this condition, a 2-D searching process (the timing metric should test any trials to find the parameter pair $(\hat{\theta}; \hat{\varepsilon})$ that maximises Equation (11)) should be performed to simultaneously search the terminal’s correct timing and carrier frequency offsets. Usually, $\theta$ and $\varepsilon$ should be searched within the range of $[-\frac{N}{2}, \frac{N}{2}]$ and the set of $\{-\frac{N}{4SF} - \Delta_a, \frac{N}{4SF} - \Delta_a, \ldots, 0, \Delta_a, \Delta_a, \ldots, \frac{N}{4SF} - \Delta_a, \frac{N}{SF} - \Delta_a\}$, respectively, where $\Delta_a > 0$ stands for the searching step in the frequency-domain. In Section 5, we will show that the maximum carrier frequency offset fine adjustment range is $|\varepsilon| < \frac{N}{2SF}$.

In order to make the remaining carrier frequency offset after an acquisition well within this fine adjustment range, $\Delta_a < \frac{N}{2SF}$ should be satisfied. The searching step $\Delta_a$ determines the carrier frequency offset acquisition accuracy. In theory, a smaller step implies a higher accuracy, but at the price of a higher complexity. Note that in the proposed carrier frequency offset fine adjustment algorithm given in Section 5, if the remaining carrier frequency offset is not beyond the fine adjustment range, the estimator will always estimate this offset with high accuracy, and that the estimation variance error is independent of the exact carrier frequency offset. Thus, extremely small $\Delta_a$ is meaningless, especially in a real system where complexity and efficiency are more important considerations than accuracy. For the sake of the tradeoff between complexity and reliability, $0.1 \leq \Delta_a \leq 0.2$ is assumed in this paper.

We compare the proposed carrier frequency offset acquisition algorithm and the Moose algorithm. A wireless communications system with a total bandwidth of 5 MHz is assumed, and a 4-tap multipath fading channel with taps attenuations of 1 dB, $-3$ dB, $-6$ dB and $-9$ dB is considered. We also assume that the carrier frequency offset is 125 kHz. For an OFDM system with a discrete Fourier transform (DFT) length of 64, the normalised carrier frequency offset is equivalent to 1.6 sub-carrier bandwidth. In order to perform the acquisition correctly, a shortened DFT length of 16 is applied in the Moose algorithm. A length-16 CP is added to this shortened training sequence to combat multipath fading. Once the remaining carrier frequency offset after acquisition is beyond the range of $[-39$ kHz, $39$ kHz], an acquisition error is assumed in the Moose algorithm. In the proposed algorithm, because we jointly perform fine frame synchronisation and carrier frequency offset acquisition, either a frame synchronisation error or a large remaining carrier frequency offset (beyond the fine adjustment range) should be seen as an acquisition error. $\Delta_a = 0.2$ is assumed in this simulation. This simulation results are illustrated in Figure 8. As the SNR increases, performance improvement will always be achieved in both algorithms, whereas the proposed acquisition scheme outperforms the Moose algorithm once SNR is higher than 6 dB. In the proposed scheme, an
additional performance improvement of about 1 dB can be achieved at low SNRs if we increase SF from 4 to 8. The low acquisition error rate in the proposed scheme guarantees its reliability in performing joint frame synchronisation and carrier frequency offset acquisition.

With a small remaining carrier frequency offset after acquisition (e.g. $\epsilon < 0.2$), $\mathbb{E}\{\Phi_9(\hat{\epsilon})\}$ will be much larger than $\mathbb{E}\{\Phi_{9-k \times SF}(\hat{\epsilon})\}$ with a high probability, which makes the probability of a timing synchronisation error negligible. Figure 9 illustrates the performance comparison between the proposed CSCL-based algorithm and that of the SS algorithm. A multipath channel of 3 taps with corresponding channel attenuations of 0 dB, −2 dB and −4 dB, respectively, is assumed. A normalised carrier frequency offset of 0.1 is applied here for both algorithms. The training sequence length for each algorithm is defined to be 256, and a length-8 CP is assumed to be added on to each data symbol. We also assume that the maximum multipath delay is smaller than the CP length. In order to mitigate ISI, the estimated training sequence start should be located with the range of $[0, CP - 1]$, where 0 is assumed to be the index of the start of the training sequence. A synchronisation error is assumed to occur once a position beyond that range maximises the timing metric. We also limit the maximum permitted PAPR to 10 dB, and peak clipping should be performed on the larger IDFT output samples. Although some power will be lost in the proposed CSCL-based algorithm after PAPR reduction, its synchronisation accuracy is still higher than that of the SS algorithm. For example, when SNR = 12 dB, a synchronisation error probability of about $6 \times 10^{-3}$ can be achieved in the SS algorithm. Better than this error, a synchronisation error probability of about $1.5 \times 10^{-3}$ can be achieved in the proposed CSCL-based algorithm when SF = 4, and it can be further reduced to about $7 \times 10^{-4}$ by increasing SF to 8.

5. CARRIER FREQUENCY OFFSET FINE ADJUSTMENT

Once the carrier frequency offset is estimated and compensated for in the acquisition stage, the remaining fractional carrier frequency offset should be further estimated/corrected with a higher accuracy. This function should resort to a carrier frequency offset fine adjustment algorithm. In many conventional fine adjustment algorithms, training sequences comprising multiple identical blocks are usually used. By estimating the phase rotations between the correlated blocks in a received training sequence, the carrier frequency offset can be estimated with a high accuracy.

5.1. Carrier frequency offset fine adjustment in the AWGN channels

The CSCL training sequence proposed in Section 4 also has a correlation between its corresponding non-zero samples,
and carrier frequency offset fine adjustment is easy to perform by estimating the phase rotations between these corresponding samples in a received training sequence. Without loss of generality, we assume that $\theta = 0$ for the sake of simplicity. If we define a received vector $y$ as $y = [y_1^T \ y_2^T]$, where $y_1^T = [y(0) \ y(SF) \cdots y(N - SF)]$ and $y_2^T = [y(N + SF - 1) \ y(2N - 1 - SF) \cdots y(2N - 1)]$, the log-likelihood function $\Lambda(\epsilon)$ can be represented as

$$\Lambda(\epsilon) = \log f(y|\epsilon)$$

$$= \log \left( \prod_{k=0}^{N/SF-1} f(y(k \cdot SF), y(2N - 1 - k \cdot SF)) \right)$$

$$= \sum_{k=0}^{N/SF-1} \log f(y(k \cdot SF), y(2N - 1 - k \cdot SF))$$

where $f(y(k \cdot SF), y(2N - 1 - k \cdot SF))$ was derived in Reference [11]. By taking the partial derivative to $\Lambda(\epsilon)$ with respect to $\epsilon$ and setting the result to zero, we have

$$\sum_{k=0}^{N/SF-1} |\Xi_k| \cdot 2\pi \epsilon(2N - 1 - 2k \cdot SF) \cdot \sin \left( \arg \{\Xi_k\} - \frac{2\pi \epsilon(2N - 1 - 2k \cdot SF)}{N} \right) = 0$$

(19)

where $\Xi_k = y^*(k \cdot SF) \cdot y(2N - 1 - k \cdot SF)$. Given a high SNR, $\arg \{\Xi_k\} - \frac{2\pi \epsilon(2N - 1 - 2k \cdot SF)}{N} \ll 1$ is satisfied for each $k$, and, therefore, we have $\sin(\arg \{\Xi_k\} - \frac{2\pi \epsilon(2N - 1 - 2k \cdot SF)}{N}) \approx \arg \{\Xi_k\} - \frac{2\pi \epsilon(2N - 1 - 2k \cdot SF)}{N}$. Based on this approximation, Equation (19) can be simplified as

$$\sum_{k=0}^{N/SF-1} |\Xi_k| \cdot (2N - 1 - 2k \cdot SF) \cdot \arg \{\Xi_k\}$$

$$= \sum_{k=0}^{N/SF-1} |\Xi_k| \cdot \frac{2\pi \epsilon(2N - 1 - 2k \cdot SF)^2}{N}$$

(20)

By solving Equation (20), an ML estimator is derived as

$$\hat{\epsilon} = \frac{N \sum_{k=0}^{N/SF-1} |\Xi_k| \cdot (2N - 1 - 2k \cdot SF) \cdot \arg \{\Xi_k\}}{2\pi \sum_{k=0}^{N/SF-1} |\Xi_k| \cdot (2N - 1 - 2k \cdot SF)^2}$$

(21)

Equation (21) requires that $|\arg \{\Xi_k\}| < \pi$ for each $k \in [0, N/SF - 1]$; i.e. $|\epsilon| < \frac{N}{2(2N - 1)}$. When $SF = 1$, this estimator reduces to that proposed in Reference [26]. Note that the proposed fine adjustment algorithm will fail if the remaining normalised carrier frequency offset after acquisition exceeds $\pm \frac{N}{2(2N - 1)}$. The frequency offset acquisition algorithm proposed in Equation (11) guarantees with a high probability that the remaining carrier frequency offset will be well within the fine adjustment range (see Figure 8). Consequently, when evaluating the performance of fine adjustment algorithms, we will always assume that the carrier frequency offset is well within the fine adjustment range.

We briefly analyse the estimation accuracy of the proposed algorithm. For each $k \in [0, N/SF - 1]$, we have $\arg \{\Xi_k\} = \frac{2\pi \epsilon_k(2N - 1 - 2k \cdot SF)}{N}$, where $\epsilon_k$ denotes the estimation error contributed by $\arg \{\Xi_k\}$, and then the estimation error of Equation (21) can be represented as

$$e = \hat{\epsilon} - \epsilon = \frac{\sum_{k=0}^{N/SF-1} |\Xi_k| \cdot (2N - 1 - 2k \cdot SF)^2 \cdot \epsilon_k}{\sum_{k=0}^{N/SF-1} |\Xi_k| \cdot (2N - 1 - 2k \cdot SF)^2}$$

(22)

From the Appendix, $\mathbb{E}[|\epsilon| |\epsilon| < \frac{N}{2(2N - 1)}; SF] = 0$; i.e. the proposed estimator is conditionally unbiased, and its variance is derived as

$$\text{Var} \left\{ e \big| |\epsilon| < \frac{N}{2(2N - 1)}; SF \right\}$$

$$\approx \frac{3 \cdot SF \cdot N \cdot \left[ \frac{1}{\pi} - \left( \frac{1}{\pi} + 1 \right) \cdot e^{-\pi} \right]}{8\pi^2[2 \cdot SF^2 + (6N - 6) \cdot SF + 4N^2 - 6N + 3]}$$

(23)

where $\pi = \frac{SF^2}{2SF^2 + \pi^2}$. At a high SNR, the Cramér–Rao lower bound (CRLB) of

$$\text{Var} \left\{ e \big| |\epsilon| < \frac{N}{2(2N - 1)}; SF \right\}$$

$$\geq \frac{3N}{4\pi^2[2 \cdot SF^2 + (6N - 6) \cdot SF + 4N^2 - 6N + 3]}$$

(24)

is met, which is derived by using the method given in Reference [29].
5.2. Carrier frequency offset fine adjustment in the Multipath channels

In this paper, Equation (21) was originally derived for an AWGN channel, and one can anticipate a performance loss in the multipath channels. This loss is proportional to the total power of the undetected low-power taps. In the multipath channel, the time-variant baseband multipath channel impulse response is modelled as in Reference [28]:

\begin{equation}
    h(t; t) = \sum_{p=1}^{N_p} \rho_p(t) \cdot \delta(t - \tau_p(t))
\end{equation}

where \(N_p\) denotes the number of multipath taps, and \(\rho_p(t)\) and \(\tau_p(t)\) are the complex amplitude and delay of the \(p\)th path, respectively. In this paper, we assume the channel impulse response does not change during one training sequence period; i.e. \(\rho_p(t) = \rho_p\) and \(\tau_p(t) = \tau_p\). Without loss of generality, we assume \(\tau_1 = 0\). A normalised wireless channel is also assumed here; i.e. \(\sum_{p=1}^{N_p} |\rho_p|^2 = 1\). For the \(p\)th tap, its average SNR is \(|\rho_p|^2 \cdot \gamma\).

In a multipath channel, if SF is not large enough, the received training sequences from different taps may interfere with each other. This inter-tap-interference reduces the effective signal-to-interference-plus-noise-ratio (SINR) of the interfered taps, and, as a result, the fine adjustment performance is degraded. The inter-tap-interference can be mitigated by letting \((SF - 1) \times T_s \geq \tau_{N_p}\) (\(T_s\) denotes the sampling period).

For a given multipath channel, we assume that a total of \(m\) taps, each with delay of \(D_1, D_2, \ldots, D_m\) samples, respectively, are detected, where \(m \times T_s < \tau_{N_p}\) and \(D_1, D_2, \ldots, D_m\) are integer values. However, in real systems, the multipath delays are usually not integer-valued, and our assumption will induce a performance loss to some extent. Because of over-sampling in real systems, this kind of performance loss is negligible if the sampling frequency is high enough. Based on Equation (21), the frequency offset fine adjustment algorithm in the multipath channel can be derived as

\begin{equation}
    \hat{\epsilon} = \frac{\sum_{p=1}^{k} |\rho_p|^2 \cdot \epsilon p}{\sum_{p=1}^{k} |\rho_p|^2}
\end{equation}

where \(1 \leq k \leq m\).

\(\epsilon_{p} = \frac{N \sum_{k=0}^{N/SF-1} |\Xi_{k,D_p}| \cdot (2N - 1 - 2k \cdot SF) \cdot \arg\{\Xi_{k,D_p}\}}{2\pi \sum_{k=0}^{N/SF-1} |\Xi_{k,D_p}| \cdot (2N-1-2k \cdot SF)^2},\)

\(\Xi_{k,D_p} = y^*(D_p + k \cdot SF)y(D_p + 2N - 1 - k \cdot SF)\)

and

\begin{equation}
    |\epsilon p|^2 \approx \frac{\sum_{k=0}^{N/SF-1} |\Xi_{k,D_p}|}{\sum_{p=1}^{k} |\Xi_{k,D_p}|}.
\end{equation}

Note that in Equation (26), \(k\) maximum power taps are used, and that a different \(k\) implies a different estimation accuracy and complexity. The estimator in Equation (26) is also conditionally unbiased, and its variance error can be derived as

\begin{equation}
    \text{Var}\{e|\epsilon| < \frac{N}{2(N-1)} \cdot SF\} = \frac{3N \cdot SF \cdot \sum_{p=1}^{k} |\rho_p|^4 \left[ \frac{1}{\pi p} - \left(1 + \frac{1}{\pi p} \right) \cdot e^{-\pi p} \right]}{8\pi^2 \left( \sum_{p=1}^{k} |\rho_p|^4 \right)^2 \left( 2 \cdot SF^2 + (6N - 6) \cdot SF + 4N^2 - 6N + 3 \right)}.
\end{equation}

where \(\Pi_p = \frac{SF^2|\rho_p|^2 \cdot \gamma}{2SF^2|\rho_p|^2 \cdot \gamma + 1}\).

6. PAPR REDUCTION CONSIDERATION

The essential advantages of the proposed algorithm over conventional algorithms result from using CSCL. However, a risk of high PAPR also results simultaneously at these non-zero (power concentrated) samples. For a given SF, the generated time-domain training symbol can be expressed as

\begin{equation}
    v = [v(0), 0, \ldots, 0, v(SF), 0, \ldots, 0, v(2 \times SF), 0, \ldots, 0, v(N - SF), 0, \ldots, 0]^T
\end{equation}

We can easily conclude that the PAPR of this training symbol cannot be smaller that SF. If we can find a frequency-domain training symbol \(x\) so as to generate a time-domain CE training symbol \(v\) with a Comb-Like shape unchanged after IDFT, we will achieve the minimum PAPR; i.e. PAPR = SF.

All training symbols that simultaneously satisfy \(|v(m \times SF)|^2 = SF^2 \cdot \frac{\gamma}{P_s}\) and \(|v(n \neq m \times SF)|^2 = 0\) will meet our minimum PAPR requirement, where \(P_s\) represents the training symbol’s total power. A representation of such a

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\)

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CE training sequence is given by

\[
v = \left[ \sqrt{\frac{SF}{N}} e^{j\alpha_0}, 0, \ldots, 0, \sqrt{\frac{SF}{N}} e^{j\alpha_1}, 0, \ldots, 0, \ldots, \sqrt{\frac{SF}{N}} e^{j\alpha_{N-1}}, 0, \ldots, 0 \right]_T
\]

(29)

where \(\alpha_k\) is a random variable uniformly distributed within \([-\pi, \pi]\). Such a training sequence is very easy to generate in a single-carrier system. If we want to generate this sequence in a multicarrier system, the modulated frequency-domain vector \(x\) is given by \(x = \text{DFT}(v)\).

The power-detection performance derived in Equation (8) is also well satisfied here. In order to perform fine frame synchronisation and carrier frequency offset estimation, as in Section 4, a central-symmetric training sequence is also utilised. Based on it, both frame synchronisation and carrier frequency offset estimation can be performed according to Equations (11) and (26), respectively. Since PAPR is reduced considerably, the requirement for a high performance amplifier can be alleviated, so that a high SF will be permitted in a real system to improve the synchronisation performance.

7. SIMULATION RESULTS

In order to compare the performances of the proposed CSCL-based algorithm (either CE or not) and the Moose algorithm [22], the DFT length is assumed to be 256, and each sub-carrier is modulated by QPSK. For a wireless signal with a total bandwidth of 10 MHz, the sub-carrier bandwidth is \(\Delta f = 39.06\) kHz. A length-16 CP is added to the front of each training sequence used in the Moose algorithm. In our simulation, the carrier frequency offset is assumed to be 3.906 kHz, which is within the estimation ranges of both algorithms. This simulation results are shown in Figures 10–16. A peak clipping will be performed to the samples of the proposed CSCL training sequence (either CE or not) that is beyond the PAPR requirement.

Although a central-symmetric training sequence performs better than the Identical-Block ones in terms of timing synchronisation, a similar advantage in performing carrier frequency offset fine adjustment in the multipath channels is realised until a large SF is used to generate its Comb-like training symbols. Figure 10 compares the performance of the proposed algorithm with SF = 1 and the Moose algorithm with respect to fine adjustment accuracy. A 3-tap multipath wireless channel with taps’ attenuations of 0 dB, \(-9\) dB and \(-12\) dB is considered here, and the first tap is used in the proposed algorithm. Even in an environment in which the maximum-power tap has most of received power, the performance of the proposed algorithm...
approaches that of the Moose algorithm only within a small SNR range of [12 dB, 18 dB]. This result occurs because in the proposed algorithm with $SF = 1$, low-power taps will act as interferences to the maximum-power tap, and, as a result, the effective SINR of the object tap is reduced.

In contrast, the performance of the Moose algorithm will not degrade too much in the multipath fading channels, provided that the CP is longer than the channel impulse response. The advantages of the proposed algorithm will be exhibited only when both the central-symmetric structure...
and comb-like shape are considered together in generating a training sequence.

In the proposed CSCL-based algorithm, when SF is smaller than the channel maximum delay spread (SF = 4 in this simulation), its performance advantage over the Moose algorithm is not apparent, as illustrated in Figure 11. The PAPR limit for the CSCL sequence is set to be 6 dB. Note that the PAPR in the Moose’s training sequence is not as high as that in the CSCL sequence, so no PAPR reduction is applied. The multipath wireless channel defined in Table 1 is used here. In the CSCL-based algorithm, when only Tap 1 is used to perform carrier frequency offset estimation, the performance may be worse than that of the Moose algorithm because of the power losses in Taps 2 to 4. In theory, carrier frequency offset estimation accuracy can be improved if more taps are used. Unfortunately, this improvement is not always achieved when SF is not large enough. Since Tap 4 acts as an interference to Tap 2, a performance floor will appear at a high SNR even if both Tap 1 and Tap 2 are utilised. Here, we should note that if only Tap 1 is used, the simulation result will match its theoretical analysis perfectly. However, the simulation and theoretical analysis will deviate if both Tap 1 and Tap 2 are utilised. This result can be explained as follows: Tap 1 is not interfered with by any other taps and can be logically seen as an AWGN channel. However, Tap 2 cannot be approximated as an AWGN channel. When N/SF is not large enough, the IDFT output cannot be well approximated as Gaussian. Without sufficient statistical information in the interference tap, the interference power cannot be totally randomised. As a result, it contributes to the large deviation to the simulation result.

In order to improve the performance of the proposed algorithm, a larger SF should be utilised. In Figure 12, SF is increased to 8. If we want to improve the proposed CSCL-based algorithm over the Moose algorithm, the PAPR limit should be as high as 10 dB. For a lower PAPR limit, e.g. 8 dB, the amount of power loss in the CSCL sequence will degrade its performance. By continuously increasing the PAPR limit to 12 dB, a performance improvement of about 0.8 dB over the Moose algorithm can be achieved in the CSCL algorithm. However, even with a high enough PAPR limit (smaller than N), the CSCL sequence will always suffer a power loss after PAPR reduction, although this loss might be negligible. For this reason, the simulation results of the CSCL algorithm cannot approach its theoretical analysis result. In order to achieve enough performance improvement, the CSCL sequence requires the PAPR of an amplifier at least as high as 10 dB. This requirement can be satisfied, especially in a multicarrier system with a large DFT length. To alleviate the PAPR requirement of amplifiers, a time domain CE training sequence is proposed to reduce the power loss after PAPR reduction. Without any power loss when the PAPR limit is 10 dB, a performance improvement over the Moose algorithm of about 1.3 dB can be achieved in the proposed CE algorithm, and the simulation result matches the theoretical analysis result perfectly. More performance improvement can be achieved if we further increase SF to 16, provided that the PAPR limit of our amplifier increases accordingly. For example, given an unchanged PAPR limit of 12 dB, a performance improvement of about 0.4 dB (or 1.7 dB) can be achieved in the proposed CSCL-based algorithm without CE modulation (or with CE modulation) over the Moose algorithm.

The SS algorithm outperforms the Schmidl algorithm [15] in both timing synchronisation and carrier frequency offset estimation. The proposed CSCL-based algorithm (either CE or not) and the SS algorithm are now compared in terms of carrier frequency offset estimation accuracy. For a fair comparison, we unify the key parameters such as the training sequence length and the sub-carrier bandwidth in both algorithms. A minor revision is made in generating the proposed CSCL training sequence: like the training sequence used by the SS algorithm, the CSCL training sequence in this simulation is also composed of one training symbol (in comparison, with the Moose algorithm, a training sequence composed of two training symbols is

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used, with the DFT length equal to one half of the training sequence length). After generating a comb-like training symbol by performing \(N\)-point IDFT (here, \(N\) equals the training sequence length), we replace the second half of that training symbol by a reverse copy of its first half. The multipath simulation environment is given in Table 1. The simulation results are illustrated in Figures 14–16. When \(SF = 4\), the performance gain of the proposed algorithm is not remarkable, and by utilizing Tap 1 only, the advantages of the CSCL algorithm over the SS algorithm become significant only at high SNR. Increasing the SF is an effective way to further improve the performance of the proposed algorithm. For example, even at low SNR, a performance advantage of about 0.3 dB over the SS algorithm in the proposed algorithm with CE modulation at low SNRs, we can set \(SF\) to 8 with a PAPR limit of 12 dB. This advantage can be further increased to about 0.5 dB by increasing \(SF\) to 16 and adjusting the PAPR limit to 15 dB. As a good tradeoff between high PAPR and high performance gain, the proposed CSCL-based algorithm with CE modulation improves performance more than that of non-CE modulation under the same PAPR constraint. For example, in order to provide a performance improvement of about 1 dB over the SS algorithm in the proposed algorithm with CE modulation at low SNRs, we can set \(SF\) to 8 with a PAPR limit of 10 dB, or set \(SF\) to 16 with a PAPR limit of 12 dB.

8. CONCLUSIONS

In this paper, a new joint frame synchronisation and carrier frequency offset estimation scheme has been proposed for burst mode multi-carrier systems. Our training sequence has a comb-like shape with a Central Symmetric structure. The shape of the proposed training symbol facilitates the detection of frames, resulting in a performance advantage over the conventional algorithms, especially at a low SNR. Based on the central-symmetric structure of the proposed training sequence, a fine frame synchronisation as well as carrier frequency offset acquisition can be jointly performed, and the maximum carrier frequency offset acquisition range is up to \(\pm \frac{N}{4SF}\) times the sub-carrier spacing. The comparisons of the proposed carrier frequency offset fine adjustment algorithm with the Moose algorithm and the SS algorithm illustrate the superior performance of the proposed algorithm with respect to estimation accuracy. This method can also be easily applied in other multicarrier systems, e.g. to perform downlink synchronisation in VSF-OFCDM, as was proposed by NTT DoCoMo.

APPENDIX

Define

\[ \mathbb{E}_k = y(2N - 1 - k \cdot SF) \cdot y^*(k \cdot SF) \]

\[ = v(2N - 1 - k \cdot SF) \cdot y^*(k \cdot SF) \cdot e^{j2\pi(2N - 1 - 2k SF)} \]

\[ + v(2N - 1 - k \cdot SF) \cdot e^{j \left( \frac{2\pi k\cdot SF}{N} + \psi \right)} \cdot w^*(k \cdot SF) \]

\[ + w(2N - 1 - k \cdot SF) \cdot y^*(k \cdot SF) \cdot e^{j \left( \frac{2\pi k\cdot SF}{N} + \psi \right)} \]

\[ + w(2N - 1 - k \cdot SF) \cdot w^*(k \cdot SF) \]

\[ = |v(k \cdot SF)|^2 \cdot e^{j2\pi(2N - 1 - 2k SF)/N} + \eta_k \]

(30)

where \(0 \leq k \leq \frac{N}{SF} - 1\), \(\eta_k\) denotes the noise-related item in Equation (30) that can be approximated as a complex Gaussian random variable with zero mean, and \(\psi\) indicates the initial phase. Estimate errors are induced by \(\eta_k\). \(R = |\eta_k|\) can be seen as a Rayleigh distributed random variable with a pdf of

\[ f(R) = \frac{R}{\sigma_{\eta_k}^2} \cdot \exp \left\{ -\frac{R^2}{2\sigma_{\eta_k}^2} \right\} \]

(31)

where \(\sigma_{\eta_k}^2 = \frac{\sigma_v^2 + \sigma_e^2}{2}\). Given the high \(\sigma_v^2\) value, we have

\[ \mathbb{E}[R] = \sqrt{\frac{\pi}{2}} \cdot \sigma_{\eta_k} \ll \sigma_v^2 \]

(32)

Define \(\alpha = \arg(\eta_k)\), which is a uniformly distributed random variable with a pdf of

\[ f(\alpha) = \frac{1}{2\pi} \cdot \alpha \in [-\pi, \pi) \]

(33)

From Equation (30), we know that \(\hat{\epsilon}_k = \epsilon + \epsilon_k = \frac{N}{2\pi} \cdot \arg(\mathbb{E}_k)\), and then

\[ \tan \left( \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot \epsilon_k}{N} \right) \]

\[ = \tan \left[ \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot (\hat{\epsilon}_k - \epsilon)}{N} \right] \]

\[ = \frac{R \cdot \sin \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot \epsilon}{N} \right)}{\sqrt{|v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot \epsilon}{N} \right)}} \]

(34)


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For small errors, \( |e_k| \ll \frac{N}{2(2N - 1 - 2k \cdot SF)} \) is satisfied, and therefore,

\[
\tan \left( \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e_k}{N} \right) = \frac{R \cdot \sin \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right)}{|v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right)} \approx \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e_k}{N} \tag{35}
\]

which results in

\[
e_k \approx \frac{N \cdot R \cdot \sin \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right)}{2\pi \cdot (2N - 1 - 2k \cdot SF) \left[ |v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) \right]} \tag{36}
\]

The expectation of \( e_k \) is given by

\[
\mathbb{E} \left\{ e_k \mid |e| < \frac{N}{2(2N - 1 - 2k \cdot SF)} \right\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{0}^{+\infty} \frac{N \cdot R \cdot \sin \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) f(\alpha) f(R)}{2\pi \cdot (2N - 1 - 2k \cdot SF) \left[ |v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) \right]} \, dR \, d\alpha
\]

\[
= 0 \tag{37}
\]

The variance of \( e_k \) is represented as

\[
\text{Var} \left\{ e_k \mid |e| < \frac{N}{2(2N - 1 - 2k \cdot SF)} \right\} = \mathbb{E} \left\{ e_k^2 \mid |e| < \frac{N}{2(2N - 1 - 2k \cdot SF)} \right\} = \frac{N^2}{4\pi^2(2N - 1 - 2k \cdot SF)^2} \int_{-\pi}^{\pi} \int_{0}^{+\infty} \frac{R^2 \cdot \sin^2 \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) f(\alpha) f(R)}{\left[ |v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) \right]^2} \, dR \, d\alpha
\]

\[
= \frac{N^2}{8\pi^3(2N - 1 - 2k \cdot SF)^2} \int_{-\pi}^{\pi} \int_{|v(k \cdot SF)|^2}^{+\infty} \frac{R^2 \cdot \sin^2 \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) f(R)}{\left[ |v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) \right]^2} \, dR \, d\alpha + \frac{N^2}{8\pi^3(2N - 1 - 2k \cdot SF)^2} \int_{-\pi}^{\pi} \int_{|v(k \cdot SF)|^2}^{+\infty} \frac{R^2 \cdot \sin^2 \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) f(R)}{\left[ |v(k \cdot SF)|^2 + R \cdot \cos \left( \alpha - \frac{2\pi \cdot (2N - 1 - 2k \cdot SF) \cdot e}{N} \right) \right]^2} \, dR \, d\alpha \tag{38}
\]
From Equation (32), we know the second part of Equation (38) is negligible, and that the variance of $e_k$ can be approximated as

$$\text{Var} \left\{ e_k \right\} \leq \frac{N^2}{8\pi^2(2N - 1 - 2k \cdot SF)^2} \approx \frac{N^2}{8\pi^2(2N - 1 - 2k \cdot SF)^2} \int_{-\pi}^{\pi} \left( R^2 \cdot \sin^2 \left( \alpha - \frac{2\pi}{N} (2N - 1 - 2k \cdot SF) \cdot e \right) \right) \frac{f(R) \, dR \, d\alpha}{\left| \varepsilon(k \cdot SF) \right|^2}$$

provided that $R \cdot \cos(\alpha - \frac{2\pi}{N} (2N - 1 - 2k \cdot SF) \cdot e) \ll \sigma_k$. Replace $\left| \varepsilon(k \cdot SF) \right|^2$ in Equation (39) with $\sigma_k^4$, Equation (39) can be further simplified as

$$\text{Var} \left\{ e_k \right\} \leq \frac{N}{2(2N - 1 - 2k \cdot SF)^2} \approx \frac{N^2}{8\pi^2(2N - 1 - 2k \cdot SF)^2} \int_{-\pi}^{\pi} \left( R^2 \cdot \sin^2 \left( \alpha - \frac{2\pi}{N} (2N - 1 - 2k \cdot SF) \cdot e \right) \right) \frac{f(R) \, dR \, d\alpha}{\left| \varepsilon(k \cdot SF) \right|^2}$$

where $\Pi = \frac{\sigma_k^2}{2\pi \epsilon} = \frac{SF^2(\gamma)^2}{2SF\gamma + 1}$.

REFERENCES

7. Atarashi SH, Sawahashi M. Variable spreading factor-orthogonal frequency and code division multiplexing (VSF-OFCDM) for broadband packet wireless access. IEICE Transactions on Communications (Japan) 2003; E86-B(1):291–299.
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